Data and control

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Apprentissage et systèmes dynamiques

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"Formulas" = data-dependent Semi Definite Programs SDPs

Dynamical control systems

We consider systems of the form

$$\dot{x} = f(x) + Bu + Ed$$

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- $d \in \mathbb{R}^q$ (disturbance)

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Systems with state-dependent input-vector fields, i.e., $\dot{x} = f(x) + g(x)u + Ed$, can be dealt with by appending a dynamical controller $\dot{u} = v$

$$\begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} f(x) + g(x)u \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} v + \begin{bmatrix} E \\ 0 \end{bmatrix} d,$$

with (x, u) the new state and v the new control input.

Assumption A $\mathcal{C}^1$ vector-valued function $Z : \mathbb{R}^n \to \mathbb{R}^s$ is known such that every entry of $f(x)$ is a linear combination of entries of $Z(x)$, i.e. $f(x) = AZ(x)$

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$Z(x)$ polynomial

Guo, De Persis, Tesi, "Data-driven stabilization of nonlinear polynomial systems with noisy data", IEEE Transactions on Automatic Control, 2022

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for some set $\mathcal{X} \subseteq \mathbb{R}^n$ and known matrix R_Q.

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If the dictionary is not complete, i.e. $f(x) = AZ(x) + d(x)$, then an analysis similar to the one to be presented carries over.

Information collection

Information about the system's dynamics is obtained from a T-long dataset of input/state samples collected during (multiple) experiment(s)

$$\mathbb{D} := \{(\dot{x}_i, x_i, u_i)\}_{i=0}^{T-1}$$

where

$$\dot{x}_i := \dot{x}(t_i), x_i := x(t_i), u_i := u(t_i)$$

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The samples satisfy

$$\dot{x}_i = AZ(x_i) + Bu_i + Ed_i, \quad i = 0, \ldots, T-1$$

A control problem

Problem Based on the dataset $\mathbb{D}$ design a state feedback controller

$$u = KZ(x)$$

that makes the closed-loop system

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exponentially contractive on $\mathcal{X}$.

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▷ The system is exponentially contractive on $\mathcal{X}$ with constant P if

$$\exists P \succ 0, \beta > 0 \colon ((A + BK)\frac{\partial Z}{\partial x})^\top P^{-1} + P^{-1}(A + BK)\frac{\partial Z}{\partial x} \preceq -\beta P^{-1} \; \forall x \in \mathcal{X}$$

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- The choice $u = KZ(x)$ allows us to express the closed-loop dynamics in terms of the dataset $\mathbb{D}$ and to reduce the design of K to a convex problem

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- No priors on feedforward control If one makes $\dot{x} = f(x) = (A + BK)Z(x) + Ed$ contractive on $\mathcal{X}$, then one is stabilising $\overline{x}(t)$ without prior knowledge of the feedforward control action $\overline{u}(t)$.

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- Regulation If the solution $\overline{x}(t)$ is not the desired one, then contractivity allows for the design of a controller that regulates (a certain function of) the state to a desired solution.

Useful data matrices

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Consider the dataset

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and store the samples into matrices U_0, X_0, X_1, Z_0 defined as

$$\begin{aligned} U_0 &:= \begin{bmatrix} u_0 & u_1 & \cdots & u_{T-1} \end{bmatrix} \\ X_0 &:= \begin{bmatrix} x_0 & x_1 & \cdots & x_{T-1} \end{bmatrix} \\ X_1 &:= \begin{bmatrix} \dot{x}_0 & \dot{x}_1 & \cdots & \dot{x}_{T-1} \end{bmatrix} \\ Z_0 &:= \begin{bmatrix} Z(x_0) & Z(x_1) & \dots & Z(x_{T-1}) \end{bmatrix} \end{aligned}$$

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Relating closed-loop trajectories to data

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Relating closed-loop trajectories to data

Consider the matrix

$$\begin{bmatrix} U_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} u_0 & u_1 & \cdots & u_{T-1} \\ Z(x_0) & Z(x_1) & \dots & Z(x_{T-1}) \end{bmatrix}$$

Given a $K \in \mathbb{R}^{m \times s}$, suppose there exists a matrix $G \in \mathbb{R}^{T \times s}$ such that

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- In the case of linear systems, $\left[\begin{smallmatrix} U_0 \\ Z_0 \end{smallmatrix}\right]$ having full row rank is guaranteed by the "input" $\left[\begin{smallmatrix} u(t) \\ d(t) \end{smallmatrix}\right]$ being persistently exciting (Willems et al's fundamental lemma).

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is obtained from off-line experiments conducted on the system

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$$P \succ 0 \tag{1a}$$

$$Z_0 \begin{bmatrix} Y_1 & G_2 \end{bmatrix} = \begin{bmatrix} P & 0_{n\times(s-n)} \\ 0_{(s-n)\times n} & I_{s-n} \end{bmatrix} \tag{1b}$$

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If it is feasible then the control law $u = KZ(x)$ with

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makes the closed-loop system exponentially contractive on $\mathcal{X}$.

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We assume to measure the state x and the regulation error

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modelling genuine noise as before.

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$$\mathcal{L} := \begin{bmatrix} d_0 \\ r_0 \end{bmatrix}, \quad M = \mathbb{1}_{1\times T}$$

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$$\mathcal{Z}_0 := \begin{bmatrix} x_0 & x_1 & \cdots & x_{T-1} \\ \xi_0 & \xi_1 & \cdots & \xi_{T-1} \\ Q(x_0) & Q(x_1) & \cdots & Q(x_{T-1}) \end{bmatrix} \qquad \mathcal{Z}_1 := \begin{bmatrix} \dot{x}_0 & \dot{x}_1 & \cdots & \dot{x}_{T-1} \\ e_0 & e_1 & \cdots & e_{T-1} \end{bmatrix}$$

$$\mathcal{L} := \begin{bmatrix} d_0 \\ r_0 \end{bmatrix}, \quad M = \mathbb{1}_{1\times T}, \quad D_0 = \begin{bmatrix} d_0 & d_0 & \dots & d_0 \\ r_0 & r_0 & \dots & r_0 \end{bmatrix}$$

Data-dependent representation of the augmented dynamics

Consider any matrices $\mathcal{K} \in \mathbb{R}^{m\times(s+p)}$, $\mathcal{G} = \begin{bmatrix}\mathcal{G}_1 & \mathcal{G}_2\end{bmatrix} \in \mathbb{R}^{T\times(s+p)}$ such that

$$\begin{bmatrix}\mathcal{K} \\ I_{s+p} \\ 0_{1\times s+p}\end{bmatrix} = \begin{bmatrix}U_0 \\ \mathcal{Z}_0 \\ M\end{bmatrix} \begin{bmatrix}\mathcal{G}_1 & \mathcal{G}_2\end{bmatrix}$$

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The matrix $\mathcal{A}+\mathcal{B}\mathcal{K}$ of the closed-loop system plant+controller

$$\begin{bmatrix}\dot{x} \\ \dot{\xi}\end{bmatrix} = (\mathcal{A}+\mathcal{B}\mathcal{K})\mathcal{Z}(x,\xi) + \mathcal{E}\begin{bmatrix}w \\ r\end{bmatrix}$$

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$$= \begin{matrix}\mathcal{A}+\mathcal{B}\mathcal{K} \\ \begin{bmatrix}\mathcal{B} & \mathcal{A} & \mathcal{E}\mathcal{L}\end{bmatrix}\end{matrix} \begin{bmatrix}\mathcal{K} \\ I_{s+p} \\ 0\end{bmatrix}$$

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The constraint

$$0 = M\mathcal{G}$$

filters out the perturbation matrix $\mathcal{E}\mathcal{L}$ from the closed-loop matrix.

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filters out the perturbation matrix $\mathcal{E}\mathcal{L}$ from the closed-loop matrix. Without it, we would obtain

$$\mathcal{A}+\mathcal{B}\mathcal{K} = (\mathcal{Z}_1 - \mathcal{E}D_0)\mathcal{G}$$

Data-dependent representation of the augmented dynamics

Consider any matrices $\mathcal{K} = \begin{bmatrix} K_x & K_\xi & K_Q \end{bmatrix} \in \mathbb{R}^{m \times (s+p)}$, $\mathcal{G} = \begin{bmatrix} \mathcal{G}_1 & \mathcal{G}_2 \end{bmatrix} \in \mathbb{R}^{T \times (s+p)}$ such that

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$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = (\mathcal{A} + \mathcal{B}\mathcal{K})\mathcal{Z}(x, \xi) + \mathcal{B}u + \mathcal{E} \begin{bmatrix} d \\ r \end{bmatrix},$$

results in

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We can then design $\mathcal{K}$ (through $\mathcal{G}$) that makes the closed-loop system contractive and – as a byproduct – returns the controller that regulates e to 0.

A formula for “tuning” controllers

A formula for "tuning" controllers

Consider the decision variables $\mathcal{P} \in \mathbb{S}^{n+p\times n+p}$, $\mathcal{Y}_1 \in \mathbb{R}^{T\times n+p}$, $\mathcal{G}_2 \in \mathbb{R}^{T\times s-n}$, $\alpha \in \mathbb{R}_{>0}$ and the SDP

$$\mathcal{P} \succ 0$$

$$\mathcal{Z}_0 \begin{bmatrix} \mathcal{Y}_1 & \mathcal{G}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{P} & 0_{n+p\times(s-n)} \\ 0_{(s-n)\times n+p} & I_{s-n} \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{Z}_1\mathcal{Y}_1 + (\mathcal{Z}_1\mathcal{Y}_1)^\top + \alpha I_n & \mathcal{Z}_1\mathcal{G}_2 & \mathcal{P}\begin{bmatrix} R_Q \\ 0_{p\times r} \end{bmatrix} \\ \star & -I_{s-n} & 0_{s-n\times r} \\ \star & \star & -I_r \end{bmatrix} \preceq 0$$

$$M \begin{bmatrix} \mathcal{Y}_1 & \mathcal{G}_2 \end{bmatrix} = 0_{1\times s+p}$$

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$$
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&\mathcal{P} \succ 0 \\
&\mathcal{Z}_0 \begin{bmatrix} \mathcal{Y}_1 & \mathcal{G}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{P} & 0_{n+p \times (s-n)} \\ 0_{(s-n) \times n+p} & I_{s-n} \end{bmatrix} \\
&\begin{bmatrix} \mathcal{Z}_1 \mathcal{Y}_1 + (\mathcal{Z}_1 \mathcal{Y}_1)^\top + \alpha I_n & \mathcal{Z}_1 \mathcal{G}_2 & \mathcal{P} \begin{bmatrix} R_Q \\ 0_{p \times r} \end{bmatrix} \\ \star & -I_{s-n} & 0_{s-n \times r} \\ \star & \star & -I_r \end{bmatrix} \preceq 0 \\
&M \begin{bmatrix} \mathcal{Y}_1 & \mathcal{G}_2 \end{bmatrix} = 0_{1 \times s+p}
\end{aligned}
$$

If it is feasible then the control law

$$
u = \mathcal{K} \begin{bmatrix} x \\ \xi \\ Q(x) \end{bmatrix} \text{ where } \mathcal{K} = U_0 \begin{bmatrix} \mathcal{Y}_1 \mathcal{P}^{-1} & \mathcal{G}_2 \end{bmatrix}
$$

makes the closed-loop system exponentially contractive on $\mathcal{X} \times \mathbb{R}^p$.

If $\mathcal{X} = \mathbb{R}^n$, then the designed controller

$$\begin{array}{rl} \dot{\xi} = & e \\ u = & \mathcal{K}\mathcal{Z}(x,\xi) \end{array} \qquad \text{where} \quad \mathcal{K} = U_0 \begin{bmatrix} \mathcal{Y}_1 \mathcal{P}^{-1} & \mathcal{G}_2 \end{bmatrix}$$

induces an equilibrium $(\,\overline{x}, \overline{\xi}\,)$ in the closed-loop system

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$$0 = (\mathcal{A} + \mathcal{B}\mathcal{K})\mathcal{Z}(\overline{x}, \overline{\xi}) + \mathcal{E} \begin{bmatrix} d \\ r \end{bmatrix},$$

such that

$$(\,\overline{x}, \overline{\xi}\,) \text{ is globally exponentially stable} \quad \underline{\text{boundedness of solutions}}$$

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$0 = \overline{e}$ — regulation

The designed regulator $\left\{ \begin{array}{rcl} \dot{\xi} = & e & \\ u = & \mathcal{K} \begin{bmatrix} x \\ \xi \\ Q(x) \end{bmatrix} & \end{array} \right.$ is a nonlinear PI controller.

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Flexibility The addition of a single constraint to the SDP ($M \begin{bmatrix} \mathcal{Y}_1 & \mathcal{G}_2 \end{bmatrix} = 0$) allows the designer to filter out a (constant) perturbation

An example - A flexible robot arm

$$\dot{x}_1 = x_2 + d_1, \quad \dot{x}_2 = -\frac{K_c}{J_2}x_1 + \frac{K_c}{J_2 N_c}x_3 - \frac{mgd}{J_2}\cos x_1 + d_2$$

$$\dot{x}_3 = x_4 + d_3, \quad \dot{x}_4 = \frac{K_c}{J_1 N_c}x_1 - \frac{K_c}{J_1 N_c^2}x_3 + \frac{1}{J_1}u + d_4$$

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Experiment

$x_0 \in [-0.1, 0.1]^4$

$u \in [-0.1, 0.1]$

$T = 10$

$$Q(x) = \cos x_1 \qquad R_Q = R_Q^\top = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{X} = \mathbb{R}^4$$

$y = x_1$ $r = \frac{\pi}{3}$

$d = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix}^\top$

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Results The solution of the SDP returns

$$\mathcal{K} = \begin{bmatrix} -3.6314 & -5.9014 & -5.1133 & -1.0990 & -0.9704 & -0.0001 \end{bmatrix}$$

in $\dot{\xi} = e, \quad u = \mathcal{K}\mathcal{Z}(x, \xi)$

An example - A flexible robot arm

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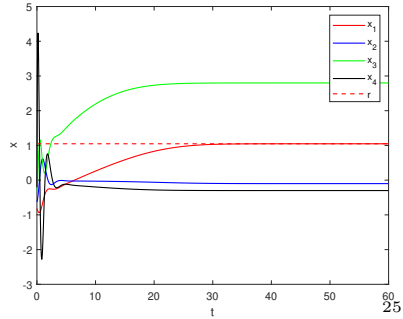
$u \in [-0.1, 0.1]$

$T = 10$

$Q(x) = \cos x_1 \qquad R_Q = R_Q^\top = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{X} = \mathbb{R}^4$

$y = x_1 \; r = \frac{\pi}{3}$

$d = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix}^\top$



Results The solution of the SDP returns

$$\mathcal{K} = \begin{bmatrix} -3.6314 & -5.9014 & -5.1133 & -1.0990 & -0.9704 & -0.0001 \end{bmatrix}$$

in $\dot{\xi} = e, \quad u = \mathcal{K}\mathcal{Z}(x, \xi)$

Conclusion

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New ways to establish systems properties and design control from training data

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Outlook

New ways to establish systems properties and design control from training data

De Persis, Tesi. "Formulas for data-driven control: stabilization, optimality, robustness". IEEE Transactions on Automatic Control, 65(3), 909-924, 2020.
De Persis, Rotulo, Tesi. "Learning controllers from data via approximate nonlinearity cancellation". IEEE Transactions on Automatic Control, 65 (3), 909-924, 2023.
Hu, De Persis, Tesi. "Enforcing contraction from data". IEEE Transactions on Automatic Control, 2025.
Hu, De Persis, Simpson-Porco, Tesi. "Data-driven harmonic output regulation of a class of nonlinear systems". Systems & Control Letters, 2025.

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New ways to establish systems properties and design control from training data

Thank you!

c.de.persis@rug.nl

Assumption A vector-valued function $Z : \mathbb{R}^n \to \mathbb{R}^s$ is known such that any entry of $Z_\star(x)$ is a linear combination of entries of $Z(x)$

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Hu, De Persis, Tesi. "Learning controllers from data via kernel-based interpolation". 62nd IEEE Conference on Decision and Control, 8509-8514, 2023.

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where we used the partition $AZ(x) = \begin{bmatrix} \overline{A} & \widehat{A} \end{bmatrix} \begin{bmatrix} x \\ Q(x) \end{bmatrix}$, $\quad CZ(x) = \begin{bmatrix} \overline{C} & \widehat{C} \end{bmatrix} \begin{bmatrix} x \\ Q(x) \end{bmatrix}$

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where $U_0 := \begin{bmatrix} u_0 & u_1 & \cdots & u_{T-1} \end{bmatrix}$

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Genuine noise

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Nonetheless, the closed-loop system is ISS with respect to $(\overline{x}, \overline{\xi})$, i.e.

$$|(\overline{x}_w(t) - \overline{x}, \overline{\xi}_w(t) - \overline{\xi})| \leq \hat{\beta}(|(x(0) - x_*, \xi(0) - \xi_*)|, t) + \hat{\gamma}(\sup_{0\leq\tau\leq t} |w(\tau)|) \quad \text{for } t \in \mathbb{R}_{\geq 0}.$$

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Hence, the regulation error $e(t)$ degrades as a function of w.

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State dependent disturbance If $\mathcal{X} = \mathbb{R}^n$, $d = d(x)$ (e.g., incomplete dictionaries) and satisfies

$$\frac{\partial d}{\partial x}(x)^\top \frac{\partial d}{\partial x}(x) \preceq R_D R_D^\top \text{ for all } x \in \mathcal{X},$$

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- Noise filtering technique

Outlook

- Control theory key technology to deal with data and dynamical control systems
- Nonlinear control design from data remains a challenging problem
- Measurement noise, I/O data, complex systems

De Persis, Tesi. "Formulas for data-driven control: stabilization, optimality, robustness". IEEE Transactions on Automatic Control, 65(3), 909-924, 2020.
De Persis, Rotulo, Tesi. "Learning controllers from data via approximate nonlinearity cancellation". IEEE Transactions on Automatic Control, 65 (3), 909-924, 2023.
Hu, De Persis, Tesi. "Enforcing contraction from data". IEEE Transactions on Automatic Control, 2025.

Conclusion

- Data-driven control of nonlinear systems expressible via "basis" functions (+ remainders)
- Design based on enforcing contractivity
- Simple end-to-end criterion (SDP⇒controller)
- Tuning of nonlinear PI control
- Noise filtering technique

Outlook

- Control theory key technology to deal with data and dynamical control systems
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Thank you!

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